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A current–current correlation function approach to hopping conductivity

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Abstract. We calculate the phonon-assisted hopping conductivity for a system of localised electronic states, using a current–current correlation function. A Green function equation-of-motion method has been applied. In doing the calculation we have established the general approximation one must make to the equations of motion and the model, in order to reproduce the random resistance network, regardless of whether one uses a position–position, a current–position, or a current–current correlation function. It is found that diagonal elements of position and electron–phonon interaction play equivalent roles and that it is insufficient to make an approximation neglecting all but diagonal position elements.

1. Introduction

Within the framework of linear response theory there are three equivalent forms which may be used for calculation of the electrical conductivity. The first arises from using an electric field as the perturbation and considers the response in the polarisation vector; this leads immediately to a position–position correlation function. The second uses an applied electric field and considers response in the current, leading to a current–position correlation function. The third form involves a current–current correlation and can be obtained from the above perturbation, or more naturally from a perturbation in the form of an applied vector potential. In terms of double time temperature-dependent Green functions (Zubarev 1960) the three forms when the perturbing field has a time dependence $e^{-i\omega t}$ are

$$\sigma(\omega) = 2\pi(ie^2\omega/\Omega)\langle\langle x|x\rangle\rangle^{\rm ret}_{E=\hbar(\omega+i\varepsilon)} \tag{1.1}$$

$$\sigma(\omega) = -2\pi e\langle\langle j_x|x\rangle\rangle^{\rm ret}_{E=\hbar(\omega+i\varepsilon)} \tag{1.2}$$

$$\sigma(\omega) = -(e/\hbar\omega)\langle[j_x, x]\rangle + (i\Omega/\omega)2\pi\langle\langle j_x|j_x\rangle\rangle^{\rm ret}_{E=\hbar(\omega+i\varepsilon)} \tag{1.3}$$

$$\langle\langle A|B\rangle\rangle_E = (1/2\pi i\hbar)\int_{-\infty}^{+\infty} dt\,\theta(t-t')\langle[A(t), B(t')]\rangle \exp[iE(t-t')/\hbar]. \tag{1.4}$$

$\langle\langle A|B\rangle\rangle$ is the retarded Green function ($\varepsilon = 0^+$), Ω is the volume of the sample, e is the

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electronic charge and $\langle\ \rangle$ is a thermal average with respect to the full Hamiltonian in the absence of a perturbing field. The equation of motion for the Green function is

$$E\langle\langle A|B\rangle\rangle_E = (1/2\pi)\langle[A, B]\rangle + \langle\langle[A, H]|B\rangle\rangle_E. \tag{1.5}$$

Our interest here is in the conductivity associated with phonon-assisted hopping of the electrons between localised electronic states in disordered solids. We therefore express our operators in a number representation based on the set of localised wavefunctions $U_\lambda(r)$ with λ labelling different sites (details may be found in Barrie *et al* (1987)):

$$h_0 U_\lambda(r) = [-(\hbar^2/2m)\nabla^2 + V(r)]U_\lambda(r) = \varepsilon_\lambda U_\lambda(r) \tag{1.6}$$

$$H = \sum_\lambda \varepsilon_\lambda a_\lambda^+ a_\lambda + \sum_q \hbar\omega_q b_q^+ b_q + \sum_{\lambda\lambda' q} B^q_{\lambda\lambda'}(b_q - b_{-q}^+)a_\lambda^+ a_{\lambda'} \tag{1.7}$$

$$B^q_{\lambda\lambda'} = \mathrm{i}g\left(\frac{\hbar q}{2V_{\mathrm{ph}}}\right)^{1/2} \int \mathrm{d}^3 r\, U_\lambda U_{\lambda'} \exp(\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}) \tag{1.8}$$

$$x = \sum_{\lambda\lambda'} x_{\lambda\lambda'} a_\lambda^+ a_{\lambda'} \tag{1.9}$$

$$j_x = (\mathrm{i}e/\Omega)\sum_{\lambda\lambda'} \omega_{\lambda\lambda'} x_{\lambda\lambda'} a_\lambda^+ a_{\lambda'} \tag{1.10}$$

$$x_{\lambda\lambda'} = \int \mathrm{d}^3 r\, U_\lambda U_{\lambda'} x \tag{1.11}$$

$$\omega_{\lambda\lambda'} = (\varepsilon_\lambda - \varepsilon_{\lambda'})/\hbar. \tag{1.12}$$

The electron–phonon interaction is assumed small and is parametrised by g in equation (1.8).

We have chosen U_λ to be real in the above. These equations lead to the following expressions for the conductivity, corresponding to equations (1.1)–(1.3):

$$\sigma(\omega) = (2\pi\mathrm{i}e^2/\Omega)\omega \sum_{\lambda\lambda'\mu\mu'} x_{\lambda\lambda'} x_{\mu\mu'} G^{\mu\mu'}_{\lambda\lambda'}(\hbar\omega + \mathrm{i}\hbar\varepsilon) \tag{1.13}$$

$$\sigma(\omega) = -(2\pi\mathrm{i}e^2/\Omega)\sum_{\lambda\lambda'\mu\mu'} \omega_{\lambda\lambda'} x_{\lambda\lambda'} x_{\mu\mu'} G^{\mu\mu'}_{\lambda\lambda'}(\hbar\omega + \mathrm{i}\hbar\varepsilon) \tag{1.14}$$

$$\sigma(\omega) = -(e/\hbar\omega)\langle[j_x, x]\rangle - (2\pi\mathrm{i}e^2/\Omega\omega)\sum_{\lambda\lambda'\mu\mu'} \omega_{\lambda\lambda'}\omega_{\mu\mu'}\, x_{\lambda\lambda'} x_{\mu\mu'} G^{\mu\mu'}_{\lambda\lambda'}(\hbar\omega + \mathrm{i}\hbar\varepsilon) \tag{1.15}$$

$$G^{\mu\mu'}_{\lambda\lambda'}(\hbar\omega + \mathrm{i}\hbar\varepsilon) = \langle\langle a_\lambda^+ a_{\lambda'}|a_\mu^+ a_{\mu'}\rangle\rangle_{\hbar\omega+\mathrm{i}\hbar\varepsilon}. \tag{1.16}$$

The primary aim of this paper is to show that all three forms lead, under appropriate approximations, to the same resistor network model for the DC conductivity, when calculated to order g^2 in the electron–phonon interaction. It transpires that this is not trivial and details are presented in § 3. The first form has previously been used by Manucharyants and Zvyagin (1974) and the second by Barrie *et al* (1987). The third form is presented here, first for completeness and secondly as a test calculation for the thermal conductivity. We plan to present a calculation of the electronic contribution to the thermal conductivity, where one typically studies an energy current–energy current correlation. Having analysed the appropriate approximation and method of calculation for the third form of the electrical conductivity, it should then be possible to extend the results presented here to the case of the thermal conductivity.

Prior to looking at the DC and relatively high-frequency cases we first look at two limits where the three methods can very easily be written in exactly the same form. The first of these is the case where g, the small parameter of electron–phonon interaction, is zero and the second is for the extremely high-frequency limit.

2. Special cases

2.1. Case of $g = 0$

For illustrative purposes, let us first discuss the case of $g = 0$, for general frequency ω. Equation (1.5) used for $G^{\mu\mu'}_{\lambda\lambda'}(E)$ leads to

$$G^{\mu\mu'}_{\lambda\lambda'}(E) = (1/2\pi)\,[(f_\lambda - f_{\lambda'})/(E + \varepsilon_\lambda - \varepsilon_{\lambda'})]\,\delta_{\lambda'\mu}\delta_{\lambda\mu'} \tag{2.1}$$

$$f_\lambda = \langle a^+_\lambda a_\lambda \rangle_{g=0}.$$

Then equation (1.13) gives

$$\sigma(\omega) = \mathrm{i}(e^2/\hbar\Omega)\sum_{\lambda\lambda'} x_{\lambda\lambda'}x_{\lambda\lambda'}\omega(f_\lambda - f_{\lambda'})\left[\mathrm{P}\left(\frac{1}{\omega + \omega_{\lambda\lambda'}}\right) - \mathrm{i}\pi\,\delta(\omega + \omega_{\lambda\lambda'})\right] \tag{2.2}$$

where P stands for the principal part. We write

$$\omega/(\omega + \omega_{\lambda\lambda'}) = 1 - \omega_{\lambda\lambda'}/(\omega + \omega_{\lambda\lambda'}) \tag{2.3}$$

and use the interchangeability of λ and λ' in the sum to show that the '1' does not contribute. This results in

$$\sigma(\omega) = -\frac{\mathrm{i}e^2}{\hbar\Omega}\sum_{\lambda\lambda'}(f_\lambda - f_{\lambda'})x^2_{\lambda\lambda'}\left[\mathrm{P}\left(\frac{\omega_{\lambda\lambda'}}{\omega + \omega_{\lambda\lambda'}}\right) - \mathrm{i}\pi\omega_{\lambda\lambda'}\delta(\omega + \omega_{\lambda\lambda'})\right]. \tag{2.4}$$

The second form for $\sigma(\omega)$, equation (1.14), leads to (2.4) directly. For the third form, equation (1.15), we use the fact that

$$\langle [j_x, x]\rangle_{g=0} = (\mathrm{i}e/\Omega)\sum_{\lambda\lambda'}\omega_{\lambda\lambda'}x^2_{\lambda\lambda'}(f_\lambda - f_{\lambda'}) \tag{2.5}$$

and combining this with the rest of the expression for $\sigma(\omega)$, we again reproduce equation (2.4).

2.2. Extremely high-frequency approximation

In hopping conductivity problems there are two frequency domains that could be described as 'high'. The limit we take in this section is the limit $\omega \gg \omega_{\lambda\lambda'}$, for a typical pair of sites. In this case the electrons behave as if they were free. However a typical value of $\omega_{\lambda\lambda'}$ is 10^{12} Hz, far above the frequency range considered as high for an applied AC signal. As an electron hops from site λ to site λ', the surrounding electrons relax with a characteristic timescale τ. In the high-frequency limit considered by Zvyagin (1980), the frequency range is such that $\tau^{-1} \ll \omega \ll \omega_{\lambda\lambda'}$.

In the case of very high frequency, we find from our equations (1.13) and (1.4) that

the first method gives

$$\sigma(\omega) = \frac{ie^2\omega}{\hbar\Omega} \sum_{\lambda\lambda'\mu\mu'} x_{\lambda\lambda'} x_{\mu\mu'} \frac{1}{(\omega + \omega_{\lambda\lambda'} + i\varepsilon)} (\delta_{\lambda'\mu}\langle a_\lambda^+ a_{\mu'}\rangle - \delta_{\lambda\mu'}\langle a_\mu^+ a_{\lambda'}\rangle)$$

$$+ (\text{terms } O(g^2/\omega)^n \text{ with } n \geqslant 1)$$

$$\sigma(\omega) \simeq \frac{ie^2\omega}{\Omega\hbar} \sum_{\lambda\lambda'\mu} x_{\lambda\lambda'} \left(1 - \frac{\omega_{\lambda\lambda'}}{(\omega + \omega_{\lambda\lambda'})}\right) (x_{\lambda'\mu}\langle a_\lambda^+ a_{\mu'}\rangle - x_{\mu\lambda}\langle a_\mu^+ a_{\lambda'}\rangle)$$

$$= -\frac{ie^2}{\hbar\omega\Omega} \sum_{\lambda\lambda'\mu} x_{\lambda\lambda'} \omega_{\lambda\lambda'} (x_{\lambda'\mu}\langle a_\lambda^+ a_\mu\rangle - x_{\mu\lambda}\langle a_\mu^+ a_{\lambda'}\rangle). \tag{2.6}$$

The second form, equation (1.14), again leads directly to equation (2.6). For the third form, equation (1.15), only the term involving $\langle [j_x, x]\rangle$ remains, in the limit as $\omega \to \infty$. We find

$$\sigma(\omega) \simeq -\frac{ie^2}{\hbar\omega\Omega} \sum_{\lambda\lambda'\mu} x_{\lambda\lambda'} (x_{\mu\lambda}\omega_{\mu\lambda}\langle a_\mu^+ a_{\lambda'}\rangle - x_{\lambda'\mu}\omega_{\lambda'\mu}\langle a_\lambda^+ a_\mu\rangle) \tag{2.7}$$

and on suitable interchanges of dummy variables λ, λ' and μ one can show that this can be written in the form (2.6).

Further interchanges of dummy variables allow us to write the common high-frequency limit as

$$\sigma(\omega) \simeq -(ie^2/\hbar\omega\Omega) \sum_{\lambda\lambda'\mu} x_{\lambda\lambda'} x_{\mu\lambda} (\omega_{\mu\lambda} - \omega_{\lambda\lambda'})\langle a_\mu^+ a_{\lambda'}\rangle. \tag{2.8}$$

In this the thermal average is with respect to the full Hamiltonian in the absence of any perturbation. Let us now check that the general result

$$\langle [j_x, x]\rangle = -i\hbar en/m \tag{2.9}$$

(n is the number density of electrons) holds for this specific case of hopping conduction. We may write

$$\langle [j_x, x]\rangle = (ie/\Omega) \sum_{\lambda\lambda'\mu} x_{\lambda\lambda'} x_{\mu\lambda} (\omega_{\mu\lambda} - \omega_{\lambda\lambda'})\langle a_\mu^+ a_{\lambda'}\rangle$$

$$= (ie/\hbar\Omega) \sum_{\lambda\lambda'\mu} [x_{\lambda\lambda'}(xh_0 - h_0x)_{\lambda\mu} - x_{\mu\lambda}(xh_0 - h_0x)_{\lambda'\lambda}]\langle a_\mu^+ a_{\lambda'}\rangle$$

$$= (ie/\hbar\Omega) \sum_{\lambda'\mu} [x, xh_0 - h_0x]_{\lambda'\mu}\langle a_\mu^+ a_{\lambda'}\rangle$$

$$= -(ie\hbar/m\Omega) \sum_{\lambda'} \langle a_{\lambda'} a_{\lambda'}\rangle. \tag{2.10}$$

Thus our common high-frequency limit for all three approaches can be written in its usual form

$$\sigma(\omega) = ine^2/m\omega. \tag{2.11}$$

3. The Green function calculation

3.1. Introduction

Let us briefly outline our reasons for claiming that the derivation of the same DC resistor model from the three forms (1.13), (1.14) and (1.15) is not trivial.

First, we note that the quadruple sum, in these equations, has to provide, in the first form, an ω^{-1} divergence; in the second, an ω-independent term; and in the third form, it has to produce a leading term proportional to ω. One could evaluate $G^{\mu\mu'}_{\lambda\lambda'}(E)$ from equation (1.5), making approximations in the process, then later perform the quadruple sum. This method proves to be rather unwieldy as it transpires that the approximations required to produce the resistor network (see § 3.2) involve, in addition to quantities appearing in the $G^{\mu\mu'}_{\lambda\lambda'}(E)$, also the coefficients in the sum. If one does solve (1.4) in the usual fashion by decoupling the hierarchy of equations stemming from it, one finds

$$
\begin{aligned}
(E+\hbar\omega_{\lambda\lambda'})G^{\mu\mu'}_{\lambda\lambda'}(E) = {} & \frac{\delta_{\lambda'\mu}}{2\pi}\Bigg[\langle a^+_\lambda a_{\mu'}\rangle - \sum_{\lambda''q} B^q_{\lambda''\lambda}\Bigg(\frac{\langle a^+_{\lambda''} a_{\mu'} b_q\rangle}{E+\hbar\omega_{\lambda''\lambda'}-\hbar\omega_q} \\
& - \frac{\langle a^+_{\lambda''} a_{\mu'} b^+_{-q}\rangle}{E+\hbar\omega_{\lambda''\lambda'}+\hbar\omega_q}\Bigg)\Bigg] - \frac{\delta_{\lambda\mu'}}{2\pi}\Bigg[\langle a^+_\mu a_{\lambda'}\rangle \\
& + \sum_{\lambda''q} B^q_{\lambda\lambda''}\Bigg(\frac{\langle a^+_\mu a_{\lambda''} b_q\rangle}{E+\hbar\omega_{\lambda\lambda'}-\hbar\omega_q} - \frac{\langle a^+_\mu a_{\lambda''} b^+_{-q}\rangle}{E+\hbar\omega_{\lambda\lambda'}+\hbar\omega_q}\Bigg)\Bigg] \\
& + \frac{1}{2\pi}\sum_q\Bigg[B^q_{\lambda'\mu}\Bigg(\frac{\langle a^+_\lambda a_{\mu'} b_q\rangle}{E+\hbar\omega_{\lambda\mu}-\hbar\omega_q} - \frac{\langle a^+_\lambda a_{\mu'} b^+_{-q}\rangle}{E+\hbar\omega_{\lambda\mu}+\hbar\omega_q}\Bigg) \\
& + B^q_{\mu'\lambda}\Bigg(\frac{\langle a^+_\mu a_{\lambda'} b_q\rangle}{E+\hbar\omega_{\mu'\lambda'}-\hbar\omega_q} - \frac{\langle a^+_\mu a_{\lambda'} b^+_{-q}\rangle}{E+\hbar\omega_{\mu'\lambda'}+\hbar\omega_q}\Bigg)\Bigg] \\
& + (\text{terms involving } G^{\mu\mu'}_{\lambda''\lambda'''}, \text{ all } \lambda'' \text{ and } \lambda'''). \qquad (3.1)
\end{aligned}
$$

The δ-functions in the inhomogeneous term (the first three lines of equation (3.1)) partly control the terms important in the quadruple sum. However, the coefficients in the sums themselves also dictate which terms are important; we assume $x_{\lambda\lambda} \gg x_{\lambda\lambda'}$, $\lambda \neq \lambda'$. Then in the first form the dominant Green function is $G^{\mu\mu}_{\lambda\lambda}$. In the second form, however, because of the presence of $\omega_{\lambda\lambda'}$, the Green functions with $\lambda = \lambda'$ do not appear in the sum. In the third form, neither $G^{\mu\mu'}_{\lambda\lambda}$ nor $G^{\mu\mu}_{\lambda\lambda'}$ appear. The fact that $\omega_{\lambda\lambda'}$ changes sign on interchange of the indices λ and λ' also mean that terms vanishing in the sum on account of oddness on interchange of indices differ in the three methods. The particular Green functions that need to be evaluated in the appropriate framework of approximations are different in the three forms.

What we have done (cf. Manucharyants and Zvyagin 1974) is to introduce the Green function $G_{\lambda\lambda'} = 2\pi\langle\langle a^+_\lambda a_{\lambda'} | S\rangle\rangle$ where S could be any operator (e.g. $S = x$ for methods (i) and (ii)). The equation (1.5) for the $G_{\lambda\lambda'}$ differs for different S only in the inhomogeneous term and is now in a convenient form to introduce the required approximations. This calculation has already been done for the first form (1.13) (Manucharyants and Zvyagin 1974) and the second form (1.14) (Barrie *et al* 1987) and has been shown to give the same result within the framework of a consistent set of approximations. We now present the calculation for the third form (1.15) and in doing so we expose the specific set of approximations required for all three calculations.

3.2. The conductivity using the $\langle\langle j_x|j_x\rangle\rangle$ Green function

Let us define the Green function $G_{\lambda\lambda'}$

$$G_{\lambda\lambda'} = 2\pi\langle\langle a_\lambda^+ a_{\lambda'} | j_x\rangle\rangle \tag{3.2}$$

($S = j_x$). Then the expression (1.3) for the conductivity can be written (note that for the sake of simplicity in the following equations, (3.2) differs from the standard Green function definition (1.4) by a factor 2π) as

$$\sigma(\omega) = -(e/\omega\hbar)\langle[j_x, x]\rangle - (e/\omega)\sum_{\lambda\lambda'} \omega_{\lambda\lambda'} x_{\lambda\lambda'} G_{\lambda\lambda'}. \tag{3.3}$$

Using equations (1.5) and (1.7) the equations of motion for $G_{\lambda\lambda'}$ are

$$(E + \hbar\omega_{\lambda\lambda'})G_{\lambda\lambda'} = (ie/\Omega)\sum_{\mu\mu'} \langle[a_\lambda^+ a_{\lambda'}, a_\mu^+ a_{\mu'}]\rangle \omega_{\mu\mu'} x_{\mu\mu'}$$
$$+ \sum_{\mu q} B^q_{\lambda'\mu}(H^-_{q\lambda\mu} - H^+_{-q\lambda\mu}) - \sum_{q\mu} B^q_{\mu\lambda}(H^-_{q\mu\lambda'} - H^+_{-q\mu\lambda'}) \tag{3.4}$$

$$(H^-_{q\lambda\mu} - H^+_{-q\lambda\mu}) = 2\pi\langle\langle a_\lambda^+ a_\mu (b_q - b^+_{-q})|j_x\rangle\rangle \tag{3.5}$$

$$(E + \hbar\omega_{mn} \mp \hbar\omega_q)H^\mp_{\pm qmn} = \sum_{\mu\mu'} (ie/\Omega)[a_m^+ a_n b_q (b^+_{-q}), a_\mu^+ a_{\mu'}]\omega_{\mu\mu'} x_{\mu\mu'}$$
$$+ \sum_{\mu q'} (B^{q'}_{n\mu} K^\mp_{qq'm\mu} - B^{q'}_{\mu m} K^\mp_{qq'\mu n}) - \sum_{ij} B^{-q}_{ij} \langle\langle a_i^+ a_j a_m^+ a_n | j_x\rangle\rangle \tag{3.6}$$

$$K^-_{qq'\lambda\lambda} = 2\pi\langle\langle a_\lambda^+ a_{\lambda'} b_q (b_{q'} - b^+_{-q'})|j_x\rangle\rangle \tag{3.7}$$

$$K^+_{qq'\lambda\lambda'} = 2\pi\langle\langle a_\lambda^+ a_{\lambda'} b^+_{-q}(b_{q'} - b^+_{-q'}|j_x\rangle\rangle. \tag{3.8}$$

The equations (3.4) and (3.6) contain all information necessary to calculate the conductivity to order g^2. However, the hierarchical series has to be truncated at equation (3.8); for this we use the following standard decoupling:

$$\langle\langle a_i^+ a_j^+ a_k a_l | j_x\rangle\rangle \simeq G_{il}\langle a_j^+ a_k\rangle + G_{jk}\langle a_i^+ a_l\rangle - G_{ik}\langle a_j^+ a_l\rangle - G_{jl}\langle a_i^+ a_k\rangle \tag{3.9}$$

$$\langle\langle a_\lambda^+ a_{\lambda'} b^+_{-q'} b_q | j_x\rangle\rangle \simeq N_q G_{\lambda\lambda'}\delta_{q+q'} \tag{3.10a}$$

$$\langle\langle a_\lambda^+ a_{\lambda'} b_{q'} b^+_{-q} | j_x\rangle\rangle \simeq (N_q + 1)G_{\lambda\lambda'}\delta_{q+q'}. \tag{3.10b}$$

In addition we have to decouple some correlation functions

$$\langle a_i^+ a_j^+ a_k a_l\rangle \simeq \langle a_i^+ a_l\rangle\langle a_j^+ a_k\rangle - \langle a_i^+ a_k\rangle\langle a_j^+ a_l\rangle \tag{3.11}$$

$$\langle a_\lambda^+ a_{\lambda'} b^+_{-q} b_{q'}\rangle \simeq \langle a_\lambda^+ a_{\lambda'}\rangle N_q \delta_{q+q'} \tag{3.12a}$$

$$\langle a_\lambda^+ a_{\lambda'} b_q b^+_{-q'}\rangle \simeq \langle a_\lambda^+ a_{\lambda'}\rangle (N_q + 1)\delta_{q+q'} \tag{3.12b}$$

$$N_q = [\exp(\beta\hbar\omega_q) - 1]^{-1}. \tag{3.13}$$

The diagonal two-point electron corelation function evaluated at $g = 0$ is

$$\langle a_\lambda^+ a_\lambda\rangle_0 = f_\lambda = [\tfrac{1}{2}\exp(\beta\varepsilon_\lambda) + 1]^{-1}. \tag{3.14}$$

(The factor $\frac{1}{2}$ is because only one electron of either spin-up or spin-down is allowed on each site.)

To find a tractable solution for the conductivity, to order g^2, and to make contact with the results given by the rate-equation approach (see review by Overhof (1976) and references therein), as distinct from the Green function approach, we make a further approximation. This is what we will refer to as the two-site approximation. After

decoupling we have a truncated hierarchy of equations (3.4), (3.5) and (3.6) which now constitute a set of equations for the G and H Green functions. On the right-hand sides of these equations we neglect all terms that involve three or more different sites. We are not restricting ourselves to only two sites; all the sites still appear in the equations but they now appear only in pairs; e.g. in (3.6) we put

$$\sum_{\mu\mu'} \langle [a_m^+ a_n b_q, a_\mu^+ a_{\mu'}] \rangle \simeq \langle [a_m^+ a_n b_q, a_m^+ a_m + a_m^+ a_n + a_n^+ a_m + a_n^+ a_n] \rangle \tag{3.15}$$

for this particular m and n.

To show now that all three forms produce the same resistor network model we use the result, valid within the framework of our approximations, that

$$B^q_{\lambda\lambda'}(x_\lambda - x_{\lambda'}) \simeq (B^q_\lambda - B^q_{\lambda'})x_{\lambda\lambda'} \tag{3.16}$$

(from now on we will abbreviate all diagonal elements by giving a single index: $B^q_{\lambda\lambda} = B^q_\lambda$, $G_{\lambda\lambda} = G_\lambda$).

Now that the approximations have been stated one does a standard Green function calculation (see Appendix) to arrive at the result

$$\sigma(\omega) = +(e^2/2\Omega) \sum_{\lambda\lambda'} (x_\lambda - x_{\lambda'}) W_{\lambda\lambda'} f_\lambda (1 - f_{\lambda'}) g_{\lambda\lambda'} \tag{3.17}$$

with

$$W_{\lambda\lambda'} = (2\pi/\hbar) \sum_q |B^q_{\lambda\lambda'}|^2 [(N_q + 1)\delta(\varepsilon_\lambda - \varepsilon_{\lambda'} - \hbar\omega_q) + N_q \delta(\varepsilon_\lambda - \varepsilon_{\lambda'} + \hbar\omega_q)] \tag{3.18}$$

and

$$g_{\lambda\lambda'} = \beta\left[x_\lambda - x_{\lambda'} + \frac{1}{(-ie\omega)}\left(\frac{G_\lambda}{\beta f_\lambda (1 - f_\lambda)} - \frac{G_{\lambda'}}{\beta f_{\lambda'}(1 - f_{\lambda'})}\right)\right]. \tag{3.19}$$

With considerable, straightforward, but tedious algebra one can calculate the following set of simultaneous equations for the diagonal Green functions

$$\omega G_\lambda = -i(-ie\omega) \sum_\mu W_{\lambda\mu} f_\lambda (1 - f_\mu) g_{\lambda\mu}. \tag{3.20}$$

The 'voltages' in the resistor network, $g_{\lambda\lambda'}$ (equation (3.19)), differ from those in methods (i) and (ii) (see equation (2.23) of Barrie *et al* (1987)) because of the factor $(-ie\omega)$ premultiplying the homogeneous part. However, the equations (3.20) are just $(-ie\omega)$ times the equivalent set in methods (i) and (ii) (equations (2.25) of Barrie *et al* (1987)). Hence at every stage of the subsequent iteration, equation (3.19) gives the same result for $\sigma(\omega)$ and the same resistor network model as methods (i) and (ii).

In this paper we have illustrated that the two-site approximation is a model-dependent, not a method-dependent, approximation and that making it leads one to the $O(g^2)$ resistor network irrespective of the path one takes. The relation (3.16), valid within the two-site approximation (Barrie *et al* 1987), provides the connecting links between the three methods. In the first method, based on expression (1.13) for the conductivity, the leading contributions to the quadruple sum have two diagonal position matrix elements and the inhomogeneous parts of the equations for the required Green functions then contain, in the two-site approximation, two off-diagonal matrix elements of the electron–phonon interaction. The second method, based on (1.14), has one off-diagonal and one diagonal matrix element of both position and electron–phonon interaction occurring in each of the leading terms, while the third method, presented here, has only off-diagonal elements of position and diagonal elements of electron–phonon interaction. In methods

(ii) and (iii), neglect of the diagonal electron–phonon interaction matrix elements (as suggested by Zvyagin 1980) would clearly be wrong.

If, in the $\omega \to 0$ limit, the terms $O(g^2/\omega)^n$ in (3.17) can be rearranged to give a g-independent contribution to the 'voltages', the expression for $\sigma(\omega)$ can be evaluated using the random resistance network model (Miller and Abrahams 1960). However, this point is currently under some debate (Barker 1976, Capek 1987). Physically these terms correspond to a relaxation δf_λ of the site occupation functions, due to the electric field (Manucharyants and Zvyagin 1974).

For finite ω ($\omega \gg \tau^{-1}$, but $\omega \ll \omega_{\lambda\lambda'}$) then the terms $O(g^2/\omega)^n$ are not singular and the diagonal Green functions give a contribution $O(g^4)$ to (3.17). This corresponds to the high-frequency limit cited by Manucharyants and Zvyagin (1974). Physically the electric field changes direction too fast for the system to equilibrate. The high-frequency expression is

$$\sigma(\omega) = (e^2\beta/2\Omega) \sum_{\lambda\lambda'} (x_\lambda - x_{\lambda'})^2 W_{\lambda\lambda'} f_\lambda (1 - f_{\lambda'}). \tag{3.21}$$

4. Conclusions

In this paper we have calculated the conductivity of a system of electrons in localised electronic states, with phonon-assisted hopping. We have used a method involving the j_x–j_x correlation function and obtained the same resistor network as the method involving x–x and that involving j_x–x. At first sight the method is an unlikely one to use for hopping conductivity because the equations involve only off-diagonal position elements. However, it is important for two reasons: it shows the resistor network model to be dependent only on a specific set of approximations, not on the particular form of the starting expression. Secondly, it can be looked upon as a test calculation for the thermal conductivity. In this calculation the natural approach would be to calculate the Green function $\langle\langle j_x^{\mathrm{E}} | j_x^{\mathrm{E}} \rangle\rangle$ where j^{E} is the energy current operator. The work presented here will shed some light on the thermal conductivity problem.

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Appendix. Evaluation of Green functions

In this appendix we present the tedious, but important, algebraic details involved in the evaluation of the required Green functions. With the stated approximations, the equations are now, for the off-diagonal Green functions,

$$(E + \hbar\omega_{\lambda\lambda'})G_{\lambda\lambda'} = -(\mathrm{i}e/\Omega)\omega_{\lambda\lambda'}\langle a_\lambda^+ a_\lambda - a_{\lambda'}^+ a_{\lambda'}\rangle x_{\lambda\lambda'} - \sum_q (B_\lambda^q - B_{\lambda'}^q)$$
$$\times (H^-_{q\lambda\lambda'} - H^+_{-q\lambda\lambda'}) + \sum_q B^q_{\lambda'\lambda}[(H^-_{q\lambda} - H^+_{-q\lambda}) - (H^-_{q\lambda'} - H^+_{-q\lambda'})] \tag{A1}$$

and for the diagonal ones

$$EG_\lambda = \sum_{q\mu} B^q_{\lambda\mu}[(H^-_{q\lambda\mu} - H^+_{-q\lambda\mu}) - (H^-_{q\mu\lambda} - H^+_{-q\mu\lambda})]. \tag{A2}$$

The decoupled equations for the H Green functions are

$$(H^{-}_{q\lambda\lambda'} - H^{+}_{-q\lambda\lambda'}) = \frac{ie}{\Omega}\omega_{\lambda\lambda'}x_{\lambda\lambda'}\Big\langle(a^{+}_{\lambda}a_{\lambda} - a^{+}_{\lambda'}a_{\lambda'})$$
$$\times\left(\frac{b_q}{E+\hbar\omega_{\lambda\lambda'}-\hbar\omega_q} - \frac{b^{+}_{-q}}{E+\hbar\omega_{\lambda\lambda'}+\hbar\omega_q}\right)\Big\rangle$$
$$- B^{-q}_{\lambda\lambda'}\left(\frac{N_q+1-f_{\lambda'}}{E+\hbar\omega_{\lambda\lambda'}-\hbar\omega_q} + \frac{N_q+f_{\lambda'}}{E+\hbar\omega_{\lambda\lambda'}+\hbar\omega_q}\right)G_{\lambda}$$
$$+ B^{-q}_{\lambda\lambda'}\left(\frac{N_q+f_{\lambda}}{E+\hbar\omega_{\lambda\lambda'}-\hbar\omega_q} + \frac{N_q+1-f_{\lambda}}{E+\hbar\omega_{\lambda\lambda'}+\hbar\omega_q}\right)G_{\lambda'}$$
$$+ \left(\frac{N_qB^{-q}_{\lambda} - (N_q+1)B^{-q}_{\lambda'}}{E+\hbar\omega_{\lambda\lambda'}-\hbar\omega_q} + \frac{(N_q+1)B^{-q}_{\lambda} - N_qB^{-q}_{\lambda'}}{E+\hbar\omega_{\lambda\lambda'}+\hbar\omega_q}\right)G_{\lambda\lambda'} \tag{A3}$$

$$(H^{-}_{q\lambda} - H^{+}_{-q\lambda}) - (H^{-}_{q\lambda'} - H^{+}_{-q\lambda'}) = -B^{-q}_{\lambda\lambda'}(N_q+1)\left(\frac{1}{E-\hbar\omega_q}+\frac{1}{E+\hbar\omega_q}\right)(G_{\lambda\lambda'} - G_{\lambda'\lambda})$$
$$+ B^{-q}_{\lambda\lambda'}(f_{\lambda} - f_{\lambda'})\left(\frac{1}{E-\hbar\omega_q} - \frac{1}{E+\hbar\omega_q}\right)(G_{\lambda\lambda'} + G_{\lambda'\lambda})$$
$$- \{B^{-q}_{\lambda}[(1-f_{\lambda'})G_{\lambda} + f_{\lambda}G_{\lambda'}] - B^{-q}_{\lambda'}[(1-f_{\lambda})G_{\lambda'} + f_{\lambda'}G_{\lambda}]\}$$
$$\times\left(\frac{1}{E-\hbar\omega_q} - \frac{1}{E+\hbar\omega_q}\right). \tag{A4}$$

We wish to find the conductivity in the first high-frequency ($\tau^{-1} \ll \omega \ll \omega_{\lambda\lambda'}$) and DC limits, hence we put (A3) into (A1) and expand to first order in ω, as we are well away from the poles of $\omega + \omega_{\lambda\lambda'}$ for a typical applied frequency ($E = \hbar\omega + i\varepsilon$).

The diagonal second-order Green functions shown above (A4) give no contribution to the conductivity to $O(g^2)$ and order ω. If one substitutes the $O(g^0)$ expression for $G_{\lambda\lambda'}$ and $G_{\lambda'\lambda}$ then each term is either exactly zero or has a symmetry on exchange of λ and λ' such that it sums to zero in the final expression (3.3). (Unless one finds an ω^{-1} divergence in dealing with the terms involving diagonal Green functions in equations (A1)–(A4), they contribute to $O(g^4)$ and so can be neglected. This is the case above for the contributing terms. This point is discussed further in the text). Thus

$$G_{\lambda\lambda'} = -(iex_{\lambda\lambda'}/\Omega\hbar)(1-\omega/\omega_{\lambda\lambda'})\langle a^{+}_{\lambda}a_{\lambda} - a^{+}_{\lambda'}a_{\lambda'}\rangle + D_{\lambda\lambda'} + I_{\lambda\lambda'} + F_{\lambda\lambda'} \tag{A5}$$

$$D_{\lambda\lambda'} = \frac{iex_{\lambda\lambda'}}{\Omega\hbar^2\omega_{\lambda\lambda'}}\left(1-\frac{2\omega}{\omega_{\lambda\lambda'}}\right)(f_{\lambda}-f_{\lambda'})\sum_q (B^q_{\lambda} - B^q_{\lambda'})$$
$$\times\left(\frac{N_qB^{-q}_{\lambda} - (N_q+1)B^{-q}_{\lambda'}}{E+\hbar\omega_{\lambda\lambda'}-\hbar\omega_q} + \frac{(N_q+1)B^{-q}_{\lambda} - N_qB^{-q}_{\lambda'}}{E+\hbar\omega_{\lambda\lambda'}+\hbar\omega_q}\right) \tag{A6}$$

$$I_{\lambda\lambda'} = +\frac{iex_{\lambda\lambda'}}{\Omega\hbar}\left(1-\frac{\omega}{\omega_{\lambda\lambda'}}\right)\sum_q (B^q_{\lambda} - B^q_{\lambda'})$$
$$\times\left\langle(a^{+}_{\lambda}a_{\lambda} - a^{+}_{\lambda'}a_{\lambda'})\left(\frac{b_q}{E+\hbar\omega_{\lambda\lambda'}-\hbar\omega_q} - \frac{b^{+}_{-q}}{E+\hbar\omega_{\lambda\lambda'}+\hbar\omega_q}\right)\right\rangle \tag{A7}$$

$$F_{\lambda\lambda'} = \frac{1}{E + \hbar\omega_{\lambda\lambda'}} \sum_{\Gamma} (B^q_\lambda - B^q_{\lambda'}) B^{-q}_{\lambda\lambda'} \left(\frac{N_q + 1 - f_{\lambda'}}{E + \hbar\omega_{\lambda\lambda'} - \hbar\omega_q} + \frac{N_q + f_{\lambda'}}{E + \hbar\omega_{\lambda\lambda'} + \hbar\omega_q} \right) G_\lambda - \frac{1}{E + \hbar\omega_{\lambda\lambda'}} \sum_q (B^q_\lambda - B^q_{\lambda'}) B^{-q}_{\lambda\lambda'} \times \left(\frac{N_q + f_\lambda}{E + \hbar\omega_{\lambda\lambda'} - \hbar\omega_q} + \frac{N_q + 1 - f_\lambda}{E + \hbar\omega_{\lambda\lambda'} + \hbar\omega_q} \right) G_\lambda. \tag{A8}$$

In going from (A1) to (A5) we moved the terms in $G_{\lambda\lambda'}$ from the RHS to the LHS, giving a mass operator of $O(g^2)$. The LHS of (A1) then becomes $(E + \hbar\omega_{\lambda\lambda'} + M_{\lambda\lambda'})G_{\lambda\lambda'}$. Multiplying LHS and RHS by $(E + \hbar\omega_{\lambda\lambda'} + M_{\lambda\lambda'})^{-1}$ and expanding to first order in g^2 and ω we arrive at (A5), with the term $D_{\lambda\lambda'}$ coming from the mass operator $M_{\lambda\lambda'}$. If we evaluate this term to order ω we find

$$\begin{aligned} D_{\lambda\lambda'} = {} & + \frac{\mathrm{i}ex_{\lambda\lambda'}}{\Omega\hbar^2\omega_{\lambda\lambda'}} (f_\lambda - f_{\lambda'}) \sum_q (B^q_\lambda - B^q_{\lambda'}) \\ & \times \frac{1}{\hbar} \left(\frac{N_q B^{-q}_\lambda - (N_q + 1) B^{-q}_{\lambda'}}{\omega_{\lambda\lambda'} - \omega_q} + \frac{(N_q + 1) B^{-q}_\lambda - N_q B^{-q}_{\lambda'}}{\omega_{\lambda\lambda'} + \omega_q} \right) \\ & - \frac{\mathrm{i}ex_{\lambda\lambda'}}{\Omega\hbar^2} \left(\frac{\omega}{\omega_{\lambda\lambda'}} \right) (f_\lambda - f_{\lambda'}) \sum_q (B^q_\lambda - B^q_{\lambda'}) \\ & \times \left[\frac{2}{\hbar\omega_{\lambda\lambda'}} \mathrm{P} \left(\frac{N_q B^{-q}_\lambda - (N_q + 1) B^{-q}_{\lambda'}}{\omega_{\lambda\lambda'} - \omega_q} + \frac{(N_q + 1) B^{-q}_\lambda - N_q B^{-q}_{\lambda'}}{\omega_{\lambda\lambda'} + \omega_q} \right) \right. \\ & - (2\mathrm{i}\pi/\omega_{\lambda\lambda'}) \{ [N_q B^{-q}_\lambda - (N_q + 1) B^{-q}_{\lambda'}] \delta(\varepsilon_\lambda - \varepsilon_{\lambda'} - \hbar\omega_q) \\ & + [(N_q + 1) B^{-q}_\lambda - N_q B^{-q}_{\lambda'}] \delta(\varepsilon_\lambda - \varepsilon_{\lambda'} + \hbar\omega_q) \} \\ & + \frac{1}{\hbar} \mathrm{P} \left(\frac{N_q B^{-q}_\lambda - (N_q + 1) B^{-q}_{\lambda'}}{(\omega_{\lambda\lambda'} - \omega_q)^2} + \frac{(N_q + 1) B^{-q}_\lambda - N_q B^{-q}_{\lambda'}}{(\omega_{\lambda\lambda'} + \omega_q)^2} \right) \\ & - \mathrm{i}\pi\beta\hbar (N_q + 1) N_q (B^{-q}_\lambda - B^{-q}_{\lambda'}) [\delta(\varepsilon_\lambda - \varepsilon_{\lambda'} - \hbar\omega_q) \\ & \left. - \delta(\varepsilon_\lambda - \varepsilon_{\lambda'} + \hbar\omega_q)] \right]. \end{aligned} \tag{A9}$$

The analytic and singular parts have been separated in the ω-dependent term above using the identity (Zubarev 1960)

$$\frac{1}{\hbar(\omega + \mathrm{i}\varepsilon + \omega_{\lambda\lambda'} \mp \omega_q)} = \frac{1}{\hbar} \mathrm{P} \left(\frac{1}{\omega + \omega_{\lambda\lambda'} \mp \omega_q} \right) - \mathrm{i}\pi\delta(\hbar\omega + \varepsilon_\lambda - \varepsilon_{\lambda'} \mp \hbar\omega_q). \tag{A10}$$

In addition we have used identities of the form

$$N_q \delta(\hbar\omega + \varepsilon_\lambda - \varepsilon_{\lambda'} - \hbar\omega_q) = [N_q - \beta\hbar\omega N_q (N_q + 1)] \delta(\varepsilon_\lambda - \varepsilon_{\lambda'} - \hbar\omega_q). \tag{A11}$$

Next let us look at the inhomogeneous term $I_{\lambda\lambda'}$. This is evaluated by expanding the density matrix, using the standard formula

$$\exp(-\beta H) = \exp(-\beta H_0) \sum_{p=0}^{\infty} \int \mathrm{d}U_1 \ldots U_p H^{\text{el-ph}}(U_1) \ldots H^{\text{el-ph}}(U_p)$$

$$\text{for } \beta > U_1 > \ldots > U_p \tag{A12}$$

$$H_0 = \sum_\lambda \varepsilon_\lambda a_\lambda^+ a_\lambda + \sum_q \hbar\omega_q b_q^+ b_q \tag{A13a}$$

$$H^{\text{el-ph}} = \sum_{\lambda\lambda' q} B^q_{\lambda\lambda'} a_\lambda^+ a_{\lambda'} (b_q - b_{-q}^+) \tag{A13b}$$

$$H^{\text{el-ph}}(U) = \exp(-UH_0) H^{\text{el-ph}} \exp(UH_0). \tag{A14}$$

Expanding to first order in $H^{\text{el-ph}}$ and using the decoupling (3.11)–(3.12) we find

$$\langle a_\lambda^+ a_\lambda b_q \rangle = \frac{1}{\hbar\omega_q}\left(B_\lambda^{-q} f_\lambda + f_\lambda \sum_{i\neq\lambda} B_i^{-q} f_i \right) \tag{A15a}$$

$$\langle a_\lambda^+ a_\lambda b_{-q}^+ \rangle = -\frac{1}{\hbar\omega_q}\left(B_\lambda^{-q} f_\lambda + f_\lambda \sum_{i\neq\lambda} B_i^{-q} f_i \right). \tag{A15b}$$

Then, using the two-site approximation, $I_{\lambda\lambda'}$ is, to order g^2 and order ω

$$\begin{aligned} I_{\lambda\lambda'} = {} & + \frac{\mathrm{i}e x_{\lambda\lambda'}}{\Omega\hbar^3} \sum_q (B_\lambda^q - B_{\lambda'}^q)\left(\frac{1}{\omega_q}\right)[B_\lambda^{-q} f_\lambda (1 - f_{\lambda'}) - B_{\lambda'}^{-q} f_{\lambda'}(1 - f_\lambda)] \\ & \times \left(\frac{1}{\omega_{\lambda\lambda'} - \omega_q} + \frac{1}{\omega_{\lambda\lambda'} + \omega_q} \right) - \frac{\mathrm{i}e x_{\lambda\lambda'}\omega}{\Omega\hbar^2\omega_{\lambda\lambda'}} \sum_q (B_\lambda^q - B_{\lambda'}^q) \\ & \times [B_\lambda^{-q} f_\lambda (1 - f_{\lambda'}) - B_{\lambda'}^{-q} f_{\lambda'}(1 - f_\lambda)] \\ & \times \left[\left(\frac{1}{\hbar\omega_q}\right) \mathrm{P}\left(\frac{1}{\omega_{\lambda\lambda'} - \omega_q} + \frac{1}{\omega_{\lambda\lambda'} + \omega_q} \right) \right. \\ & - (2\mathrm{i}\pi/\omega_{\lambda\lambda'}) [\delta(\varepsilon_\lambda - \varepsilon_{\lambda'} - \hbar\omega_q) - \delta(\varepsilon_\lambda - \varepsilon_{\lambda'} + \hbar\omega_q)] \\ & \left. + \left(\frac{\omega_{\lambda\lambda'}}{\omega_q}\right) \mathrm{P}\left(\frac{1}{(\omega_{\lambda\lambda'} - \omega_q)^2} + \frac{1}{(\omega_{\lambda\lambda'} + \omega_q)^2} \right) \right]. \end{aligned} \tag{A16}$$

Finally we have to deal with the homogeneous term $F_{\lambda\lambda'}$. For this we are only interested in the contribution that diverges as ω^{-1}. This point is discussed in more detail in the text; let it suffice at present to say that the analytic contribution to $F_{\lambda\lambda'}$ is of order g^4 and can be neglected. Thus we are able to set $\omega = 0$ in calculating $F_{\lambda\lambda'}$. After some algebra $F_{\lambda\lambda'}$ can be written

$$\begin{aligned} F_{\lambda\lambda'} = {} & - \frac{\mathrm{i}\pi}{\hbar\omega_{\lambda\lambda'}} \sum_q (B_\lambda^q - B_{\lambda'}^q) B_{\lambda\lambda'}^{-q} \\ & \times \left(\frac{(1 - f_{\lambda'})}{(1 - f_\lambda)} [(N_q + 1)\delta(\varepsilon_\lambda - \varepsilon_{\lambda'} - \hbar\omega_q) + N_q \delta(\varepsilon_\lambda - \varepsilon_{\lambda'} + \hbar\omega_q)] G_\lambda \right. \\ & \left. - \frac{(1 - f_\lambda)}{(1 - f_{\lambda'})} [N_q \delta(\varepsilon_\lambda - \varepsilon_{\lambda'} - \hbar\omega_q) + (N_q + 1)\delta(\varepsilon_\lambda - \varepsilon_{\lambda'} + \hbar\omega_q)] G_{\lambda'} \right) \\ & + \frac{1}{\hbar^2\omega_{\lambda\lambda'}} \sum_q (B_\lambda^q - B_{\lambda'}^q) B_{\lambda\lambda'}^{-q} \left[\mathrm{P}\left(\frac{N_q + 1 - f_{\lambda'}}{\omega_{\lambda\lambda'} - \omega_q} + \frac{N_q + f_{\lambda'}}{\omega_{\lambda\lambda'} + \omega_q} \right) G_\lambda \right. \\ & \left. - \mathrm{P}\left(\frac{N_q - f_\lambda}{\omega_{\lambda\lambda'} - \omega_q} + \frac{N_q + f_\lambda - 1}{\omega_{\lambda\lambda'} + \omega_q} \right) G_{\lambda'} \right]. \end{aligned} \tag{A17}$$

Equalities of the form

$$(N_q + 1 - f_{\lambda'})\delta(\varepsilon_\lambda - \varepsilon_{\lambda'} - \hbar\omega_q) = \frac{(1 - f_{\lambda'})}{(1 - f_\lambda)}(N_q + 1)\delta(\varepsilon_\lambda - \varepsilon_{\lambda'} - \hbar\omega_q) \tag{A18}$$

have been used in equation (A17).

Now, with equations (A9), (A16) and (A17) we have an expression for $G_{\lambda\lambda'}$ (equation (A5)) which we can use in equation (3.3) for the conductivity. The analytic terms in these expressions disappear in the final equation (3.3) for $\sigma(\omega)$ because of the symmetry on interchange of λ and λ'. Singular terms of order ω, one from (A9) and one from (A16) cancel each other because

$$\begin{aligned}(f_\lambda - f_{\lambda'})\{&[(N_q + 1)B_\lambda^{-q} - N_q B_{\lambda'}^{-q}]\delta(\varepsilon_\lambda - \varepsilon_{\lambda'} - \hbar\omega_q) \\ &+ [N_q B_\lambda^{-q} - (N_q + 1)B_{\lambda'}^{-q}]\delta(\varepsilon_\lambda - \varepsilon_{\lambda'} + \hbar\omega_q)\} \\ &= -[B_\lambda^{-q} f_\lambda(1 - f_{\lambda'}) - B_{\lambda'}^{-q} f_{\lambda'}(1 - f_{\lambda'})] \\ &\times [\delta(\varepsilon_\lambda - \varepsilon_{\lambda'} - \hbar\omega_q) - \delta(\varepsilon_\lambda - \varepsilon_{\lambda'} + \hbar\omega_q)]\end{aligned} \tag{A19}$$

and the final expression for the conductivity is

$$\begin{aligned}\sigma(\omega) = {} &+(e^2/2\Omega)\sum_{\lambda\lambda'}(x_\lambda - x_{\lambda'})W_{\lambda\lambda'} f_\lambda(1 - f_{\lambda'})g_{\lambda\lambda'} + (ie^2/\Omega\hbar\omega) \\ &\times \sum_{\lambda\lambda'} \omega_{\lambda\lambda'} x_{\lambda\lambda'}(x_\lambda - x_{\lambda'})\langle a_\lambda^+ a_{\lambda'}\rangle \\ &- (ie^2/\Omega\hbar^3\omega)\sum_{\lambda\lambda' q} x_{\lambda\lambda'}^2 (B_\lambda^q - B_{\lambda'}^q)(f_\lambda - f_{\lambda'})\left(\frac{N_q B_\lambda^{-q} - (N_q + 1)B_{\lambda'}^{-q}}{\omega_{\lambda\lambda'} - \omega_q}\right. \\ &\left. + \frac{(N_q + 1)B_\lambda^{-q} - N_q B_{\lambda'}^{-q}}{\omega_{\lambda\lambda'} + \omega_q}\right) - \frac{ie^2}{\Omega\hbar^3\omega}\sum_{\lambda\lambda' q} x_{\lambda\lambda'}^2 (B_\lambda^q - B_{\lambda'}^q) \\ &\times \left(\frac{\omega_{\lambda\lambda'}}{\omega_q}\right)[B_\lambda^{-q} f_\lambda(1 - f_{\lambda'}) - B_{\lambda'}^{-q} f_{\lambda'}(1 - f_{\lambda'})] \\ &\times \left(\frac{1}{\omega_{\lambda\lambda'} - \omega_q} + \frac{1}{\omega_{\lambda\lambda'} + \omega_q}\right)\end{aligned} \tag{A20}$$

with $W_{\lambda\lambda'}$ and $q_{\lambda\lambda'}$ given in the text. In arriving at (A19) we have invoked equation (3.16).

If one calculates $\langle a_\lambda^+ a_{\lambda'}\rangle$ to order g^2 using equation (A12) one finds the sum involving the correlation function exactly cancels the last two sums in equation (A20) and one is left with a very familiar result (3.17).

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